Cepheid Mass-loss and the Pulsation – Evolutionary Mass Discrepancy

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ABSTRACT

I investigate the discrepancy between the evolution and pulsation masses for Cepheid variables. A number of recent works have proposed that non-canonical mass-loss can account for the mass discrepancy. This mass-loss would be such that a $5M_\odot$ star loses approximately 20% of its mass by arriving at the Cepheid instability strip; a $14M_\odot$ star, none. Such findings would pose a serious challenge to our understanding of mass-loss. I revisit these results in light of the Padova stellar evolutionary models and find evolutionary masses are $(17 \pm 5)\%$ greater than pulsation masses for Cepheids between $5 < M/M_\odot < 14$. I find that mild internal mixing in the main-sequence progenitor of the Cepheid are able to account for this mass discrepancy.

Subject headings: Cepheids:pulsation stellar:evolution

1. Introduction

δ Cepheid variables are an essential step in our determination of extragalactic distances. Apart from their use as distance indicators, the regularity of Cepheid pulsation provides a well-defined set of observational parameters with which to probe the course of stellar evolution for intermediate mass stars.

Over the preceeding decades much effort has been devoted to reconciling mass determinations for Cepheids from the various methods at our disposal (see Cox (1980) for a review). The longest standing of these, the Cepheid pulsation mass discrepancy was first revealed by Stobie (1969) who showed that pulsation masses for bump Cepheids[1] were significantly

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[1]Bump Cepheids are distinguished by a secondary local maximum of the light curve seen in Cepheids with periods in the range of 6 to 16 days (Bono et al. 2000b; Hertzsprung 1926).

lower than those predicted by stellar evolutionary models. Andreasen (1988) showed that by artifically enhancing the opacity over the temperature range $1.5 \times 10^5 K < T < 8 \times 10^5 K$ by a factor of 2.5 it was possible to remove the discrepancy. More detailed modelling of opacities by the OPAL (Rogers & Iglesias 1992) and the Opacity Project (Seaton et al. 1994) confirmed an increase in this temperature range due to metal opacity.

The implementation of new opacities largely resolved the bump Cepheid mass discrepancy (Moskalik et al. 1992). However, despite convergence, a number of subsequent studies have shown that the discrepancy remains significant and requires explanation. Studies of Galactic (Natale et al. 2007; Caputo et al. 2005), LMC (Wood et al. 1997; Keller & Wood 2002, 2006; Bono et al. 2002) and SMC (Keller & Wood 2006) Cepheids have shown that the masses determined via pulsation modelling are ∼15-20% less massive than those expected from evolutionary models. Dynamical masses for Cepheids are difficult to obtain given the low spatial density of Cepheids. The works of Benedict et al. (2007); Evans et al. (2007, 2006, 1998) present dynamical masses for six Cepheids. Albeit with large associated uncertainties, these results confirm the conclusions drawn for pulsation modelling; that the evolutionary masses appear ∼ 15% larger.

From an evolutionary perspective, Cepheids are understood to be post-red giant stars crossing the instability strip on so-called blue loops following the initiation of core-He burning. To ascribe an evolutionary mass one takes the Cepheid's luminosity and, using a mass-luminosity (M-L) relation that is derived from evolutionary models, derives the mass of the Cepheid. The M-L relation can be modified substantially by the treatment of internal mixing and mass-loss. Both processes feature complex hydrodynamical and radiative mechanisms for which we, at present, only possess empirical approximations.

The treatment of internal mixing modifies the size of the helium core established during the star's main-sequence (MS) evolution. Overshoot at the edge of the convective core of the Cepheid progenitor mixes additional hydrogen into the core and hence increases the helium core mass. As a consequence the post-MS evolution occurs at a higher luminosity. Mass loss, in an ad-hoc manner at least, offers a mechanism to modify the M-L relation by directly reducing the mass of a Cepheid.

The properties of pulsation, on the other hand, are dependent on the structure of the atmosphere of the Cepheid. Keller & Wood (2006) show that the morphology of a bump Cepheid light curve is highly sensitive to the mass, luminosity, effective temperature and metallicity. Hence modelling of the light curve can be used to determine a pulsation Cepheid mass that is entirely independent of stellar evolution calculations.

In the work of Bono et al. (2002) and Caputo et al. (2005) it is proposed that mass-

loss can account for the mass discrepancy between pulsation and evolutionary masses. Caputo et al. (2005) also conclude that models that incorporate additional internal mixing in the vicinity of the convective core are not able to explain the mass discrepancy. In this paper we revisit these conclusions and present a scenario for resolution of the mass discrepancy.

2. The Cepheid Pulsation – Evolutionary Mass Discrepancy

A useful way of expressing the mass discrepancy is to form the quantity $\Delta M/M_E$ in which ΔM is the difference between the pulsation mass, M_P, and the canonical (i.e. not including CCO) evolutionary mass, M_E, as shown in Figure 1. In Figure 1 we also show the effects of the inclusion of mild (Λ_c=0.5) and moderate (Λ_c=1.0) CCO. The effect of CCO is to raise the luminosity of the Cepheid of a given mass as discussed above.

Caputo et al. (2005) utilise the $BVIJK$ absolute magnitudes derived by Storm et al. (2004) using distances from the near-infrared surface brightness (IRSB) method. Caputo et al. (2005) have used the mass dependent period-luminosity-color relation to determine the pulsational Cepheid mass using the M-L relation of Bono et al. (2000a). They find that $\Delta M/M_E$ ranges from 20% at $M \sim 5 M_\odot$ to $\sim$0% at $M \sim 14 M_\odot$ (see Figure 1). Such a trend can not be explained by a uniform level of core overshoot. Monte-Carlo simulation using the individual quoted uncertainties of the data shown in Figure 1 reveals a gradient of $(3.4 \pm 0.5)\% M_\odot^{-1}$.

A caveat to IRSB analysis that underlies the results of Storm et al. (2004) and Caputo et al. (2005) is the necessary introduction of the poorly understood projection factor, p, that embodies the effects of limb-darkening (Nardetto et al. 2006), and is dependent on the pulsation velocity and the species under study (Nardetto et al. 2007). p is approximated as a function of period, and it has been speculated that this could introduce a period dependency in the derived distances. However, the work of Fouque et al. (2007) shows that direct HST parallaxes for a sample of Cepheids agree within uncertainties with the IRSB measures albeit for only the five stars with distance determinations from both techniques.

To explain the mass discrepancy found at lower masses, Caputo et al. (2005), propose mass-loss from the Cepheid progenitor before, or during the central He-burning phase. The implied total mass-loss *declines* from $\sim 20\%$ at $M \sim 5 M_\odot$ and vanishes by $M \sim 14 M_\odot$. Such mass-loss is seemingly at odds with empirical estimates of mass-loss rates which show that mass-loss *increases* with stellar luminosity and radius (Reimers 1975; de Jager et al. 1988; Schroeder & Cuntz 2007). Furthermore, Caputo et al. conclude that CCO can not account for this trend in $\Delta M/M_E$ with mass since this would lead to unphysical $\Delta M/M_{E,\Lambda} < 0$

(their Figure 9; where $\Delta M/M_{E,\Lambda}$ is the mass at a given Λ_c) for higher mass Cepheids.

The Cepheid M-L relation implemented in analysis is critical. In their study, Caputo et al. (2005) have chosen to utilise a linear M-L relation derived from Bono et al. (2000a) evolutionary models (Z=0.02, Y=0.28 and Λ_c=0) that do not incorporate mass loss. This M-L relation is shown as the dashed line in Figure 2. However, as shown by the evolutionary models of Bono et al. (2000a) there are significant departures from this linear relationship for $M \geq 9M_\odot$ (see the dotted line in Figure 2).

In the analysis to follow we incorporate the non-linear nature of the Cepheid M-L into our analysis. The models of (Bono et al. 2000a, Z=0.02, Y=0.27 and $\Lambda_c = 0$) extend to M=12$M_\odot$, however the derived pulsation masses reach to 14.9$M_\odot$. Let us consider the stellar evolutionary sequences of Bressan et al. (1993) that extend to higher masses. These models (Z=0.02, Y=0.28 and Λ_c=0) implement mass-loss according to the Reimers (1975) and de Jager et al. (1988) formulation and are shown in Figure 2 as the solid line. For masses less than 8$M_\odot$ there is clearly a different gradient between the Bono et al. (2000a) and Bressan et al. (1993) M-L, a feature discussed by Beaulieu et al. (2001). What is common to both models is a marked departure from a linear M-L in the mass range 9-10$M_\odot$. This departure results in evolutionary masses that are greater for a given luminosity compared with those derived from the linear M-L for masses greater than 9-10$M_\odot$.

The use of the Bressan et al. (1993) M-L results in Figure 3. Figure 3 reveals no significant trend in $\Delta M/M_E$ as a function of M_E (Monte-Carlo simulation of the data and associated uncertainties shown in Figure 3 reveals a gradient of $(0.3 \pm 0.5)\% M_\odot^{-1}$). This demonstrates that a consideration of the non-linear Cepheid M-L accounts for the decrease in $\Delta M/M_E$ seen in the work of Caputo et al. The absence of a significant gradient also negates the argument by Caputo et al. (2005) that CCO can not be the source of the Cepheid mass discrepancy.

Reanalysis of the results of Caputo et al. (2005) shows that the evolutionary Cepheid mass is $(17 \pm 5)\%$ greater than that predicted from pulsation modelling. In the next section I discuss the merits of a range of possible causes.

3. The Source of the Cepheid Pulsation – Evolutionary Mass Discrepancy

The hiding place of the source of discrepancy between pulsation and evolutionary Cepheid masses has shrunk dramatically over the last three decades. To account for the discrepancy we have three key options. Firstly, it is possible that some input physics in pulsation calculations are not sufficiently described. In this regard, the only input that could

affect pulsation to this magnitude is the description of radiative opacity. Second, that the Cepheid mass is smaller than their main-sequence progenitors due to mass-loss. And finally, that evolutionary calculations underestimate the mass of the He core in intermediate mass stars and so underestimate their luminosity. We now discuss each of these possible sources in detail.

3.1. Radiative Opacity

The pulsation properties of Cepheids are critically dependent on the so-called Z-bump opacity arising from the dense spectrum of transitions originating from highly ionized Fe. The inclusion of these transitions in the works of OPAL (Rogers & Iglesias 1992) and OP (Seaton et al. 1994) resulted in a substantial increase in opacity at $\log T \approx 5.2$. The Opacity Project (Badnell et al. 2005) has included further details of atomic structure (in particular, the treatment of atomic inner shell processes) in their calculation of opacity. The new opacities do show an increase over the 1992 OP and OPAL values of opacity in the Z-bump, but at a level of only 5-10% (Badnell et al. 2005). To account for the mass discrepancy the opacity would need to be raised by 40-50%, equivalent to the increase between the early Los Alamos opacities (Cox & Tabor 1976) and OP and OPAL opacities. Hence the uncertainty in radiative opacity is an unlikely resolution to the Cepheid mass discrepancy.

3.2. Mass-Loss

The studies of Bono et al. (2002, 2006) and Caputo et al. (2005) propose that mass-loss can account for the mass discrepancy. Candidate mass-loss phases include the red giant branch phase, subsequent blue loop evolution, or possibly from the action of pulsation itself (Bono et al. 2006). The removal of mass from the Cepheid is a straightforward, albeit ad-hoc, way to bring the evolutionary mass in line with that derived from pulsation. The timing of, and the changes in stellar structure brought about by significant mass-loss are important to the net change in the Cepheid M-L. For instance, significant mass-loss on the MS causes a reduction in overall mass and hence helium core mass, resulting in a reduction in luminosity in the instability strip (de Loore 1988). Enhanced mass-loss on the red giant branch reduces envelop mass, and the material available to the hydrogen-burning shell within the Cepheid, again leading to a reduction in luminosity relative to a star without mass-loss (Yong et al. 2000). One of the difficulties with the proposal for mass-loss to solve the Cepheid mass discrepancy is that it would require the rather artificial bulk removal of material without consequent modifications to stellar structure and energy production.

Furthermore, standard mass-loss can account for at most a few percent reduction in Cepheid mass and not the 15%-20% required. Mass loss is usually treated the semi-empirical relation of Reimers (1975, 'Reimers' law'). While not providing any physical reasoning on why the mass-loss is generated, 'Reimers' law' provides an adequate match to observed mass-loss rates over a broad range of stellar parameters (Schroeder & Cuntz 2007).

Major mass-loss is expected during the red giant branch (RGB) evolution. The models of Girardi et al. (2000) and Bressan et al. (1993) use a parameterised, empirical fit $dM/dt = -4 \times 10^{-13} \eta L/gR$ (Reimers 1975). The value of η is set by a consideration of the masses of stars on the horizontal branch (HB) of globular clusters. Determination of η is made difficult due to the variety of HB morphology exhibited by globular clusters. If we consider the distribution of effective temperatures for HB stars is entirely due to variable mass-loss then η must range from 0 to somewhat more than 0.4. Using the canonical value of $\eta = 0.4$, a $5M_\odot$ star looses $\sim 0.03M_\odot$ during the RGB phase. To accomodate the mass discrepancy seen in Figure 1 this would have to be increased to $0.8M_\odot$ corresponding to a 20-30 fold increase in η which is not plausible. Therefore mass-loss on the RGB does not resolve the mass discrepancy.

Caputo et al. (2005) and Bono et al. (2006) suggest that pulsation may give rise to an enhancement of mass-loss. Attempts to measure the mass-loss rates for Cepheids have been made using IRAS infrared excesses (McAlary & Welch 1986) and IUE UV line profiles (Deasy 1988) and in the radio (Welch & Duric 1988). Deasy (1988) found mass-loss rates for the majority of Cepheids of the order of a few $\times 10^{-9}$ $M_\odot \mathrm{yr}^{-1}$. The study of Welch & Duric (1988) places upper limits on the mass-loss rate of $<10^{-7}$ $M_\odot \mathrm{yr}^{-1}$. Welch & Duric (1988) and Deasy (1988) conclude that mass-loss during the Cepheid phase is insufficent to explain the observed mass discrepancy. In the case of a $5M_\odot$ Cepheid, mass loss can account for the mass discrepancy if either the lifetime in the IS is ten times longer than derived by Bono et al. (2000a) or mass loss is thirty times greater than found by Deasy (1988). Mass loss as an explanation for the mass discrepancy in a $12M_\odot$ Cepheid is more challenging. The mass loss found by Deasy over the entire lifetime of the star can not account for the mass discrepancy and a 600-fold increase in the mass loss rate would be required.

Recently, Mérand et al. (2007) found using near-infrared interferometry that, from a sample of four Cepheids all show the presence of some circumstellar material. α Persei, a non-variable supergiant residing in the instability strip, does not show evidence for circumstellar material, suggesting that pulsation does have a role in enhancing mass-loss. The conversion from historical and ongoing mass loss that lead to the circumstellar material and the rate of mass loss is beyond the scope of the study of Mérand et al. (2007) however. On the current evidence I conclude that mass loss does not present a solution to the Cepheid mass

discrepancy.

3.3. Core He Mass and Internal Mixing

Cepheid luminosity is critically dependent on the He core mass. The mass of the He core is determined by the extent of the convective core during core H burning. Classical models define the limit to convection via the Schwarzschild criterion. This places the boundary to convection at the radius at which the buoyant force acting on a hot clump of material rising from the convective core drops to zero. However, the temperature and density regime in the vicinity of the convective boundary of the main sequence Cepheid progenitor are such that restorative forces in the region formally stable to convection are mild, and some significant level of overshoot of the classical boundary is expected (Zahn 1991; Deng & Xiong 2007).

The description of convection is the weakest point in our understanding of the physics of intermediate mass – massive stars. Numerical modeling of core convection requires a description of the turbulence field at all scales. Three-dimensional hydrodynamical calculations capable of adequate resolution have only recently become a possibility and are in their infancy (Meakin & Arnett 2007; Dearborn et al. 2006; Eggleton et al. 2003, 2007). In the absence of a general theory of CCO, which would enable us to calculate the amount of core overshoot for a star of a given mass and chemical composition, a semiempirical phenomenological approach must be used in calculations of stellar evolution. Several computational schemes of various degrees of sophistication in treating the physics of overshoot have been discussed in the literature (Maeder & Meynet 1988; Bertelli et al. 1990; Girardi et al. 2000; Demarque et al. 2007; Straka et al. 2005). The most common parameterization of core overshoot utilizes the mixing-length formulation, where gas packets progress a distance of Λ_c[2] pressure scale heights into the classically stable region. Λ_c offers a convenient way to parametrize the extension of the convective core it does not constrain its physical origin. In addition to the CCO mechanism outlined above, rotationally induced mixing can similarly be invoked to bring about a similar range of internal mixing. As shown by Heger & Langer (2000) and Meynet & Maeder (2000) mixing in the sheer layer formed at the interface between the convective and radiative regions can lead to larger He core masses for massive stars.

At present we must rely on observation for constraint of Λ_c. Observational determi-

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[2]We quantify core overshoot using the formalism of Bressan et al. (1981). Note that Λ_c is a factor of 2.0 times the overshoot parameter d_{over}/H_p in the formalism of the Geneva group (Schaller et al. 1992; Demarque et al. 2004)

nations have focussed on populations of intermediate-mass stars (M=5-12$M_\odot$), where the signature of CCO is expected to be most clearly seen. The studies of Mermilliod & Maeder (1986) and Chiosi et al. (1992) of Galactic open clusters converge on the necessity for mild core overshoot; $\Lambda_c \approx 0.5$. A number of subsequent studies have presented evidence both for (Barmina et al. 2002) and against (Testa et al. 1999) core overshoot. From studies of the young populous cluster systems of the Magellanic Clouds, Keller et al. (2001) found evidence for $\Lambda_c = 0.62 \pm 0.11$, while Cordier et al. (2002) examined the Magellanic Cloud field population and found the necessity for a level of overshoot between 0.2 and 0.8 pressure scale heights. Measurements with the lowest associated uncertainties are derived from the pulsation modelling of Cepheid light curves by Keller & Wood (2002, 2006); Bono et al. (2002). Keller & Wood (2006) finds evidence for a weak dependence of Λ_c with metallicity with Λ_c rising from 0.688±0.009 in the LMC (metallicity 2/5th Solar) to 0.746±0.009 in the SMC (metallicity 1/5th Solar).

While radiative opacity and mass-loss have proven insufficient, increased internal mixing remains as the most likely cause of the Cepheid mass discrepancy. The excess of (17±0.05)% in evolutionary mass compared to the pulsation mass equates to a uniform level of CCO of $\Lambda_c = 0.67 \pm 0.17$. This degree of overshoot is within the range of previous studies discussed above. The scenario I have outlined offers a straightforward explanation for the findings of Caputo et al. (2005), one that uses canonical mass-loss and mild convective core overshoot.

4. Conclusions

In this paper I revisit the conclusions of Caputo et al. (2005) regarding the cause of the observed discrepancy between the evolution and pulsation masses for Cepheids. Caputo et al. (2005) find that Cepheids of $5M_\odot$ have 20% less mass, as determined from pulsation analysis, than expected from evolutionary calculations, while the evolution and pulsation masses for Cepheids of $14M_\odot$ agree. In order to explain this finding Caputo et al. propose a scenario of non-standard mass-loss:- one that sees increased mass-loss at lower masses and drops to negligible mass-loss at $M \geq 14M_\odot$. This scheme of mass-loss would be counter to the observationally grounded evidence that accumulated mass-loss by the epoch of core-He burning increases with increasing stellar mass. Furthermore, Caputo et al. claim that convective core overshoot is unable to provide a solution as it can not account for the trend of mass discrepancy with Cepheid mass.

The findings of Caputo et al. are based on a Cepheid mass-luminosity relationship that proves erroneous when extrapolated to higher masses. In this paper I show that including a full description of the mass-luminosity relation results in a mass discrepancy in which evolu-

tionary masses are $(17 \pm 5)\%$ greater than pulsation masses. The trend of mass discrepancy with Cepheid mass is removed.

I propose that additional internal mixing, as parametrised in the convective core overshooting paradigm, is the primary mechanism giving rise to the mass discrepancy. The level of convective core overshoot so derived is $\Lambda_c = 0.67 \pm 0.17$, a value which agrees with previous determinations from a range of techniques.

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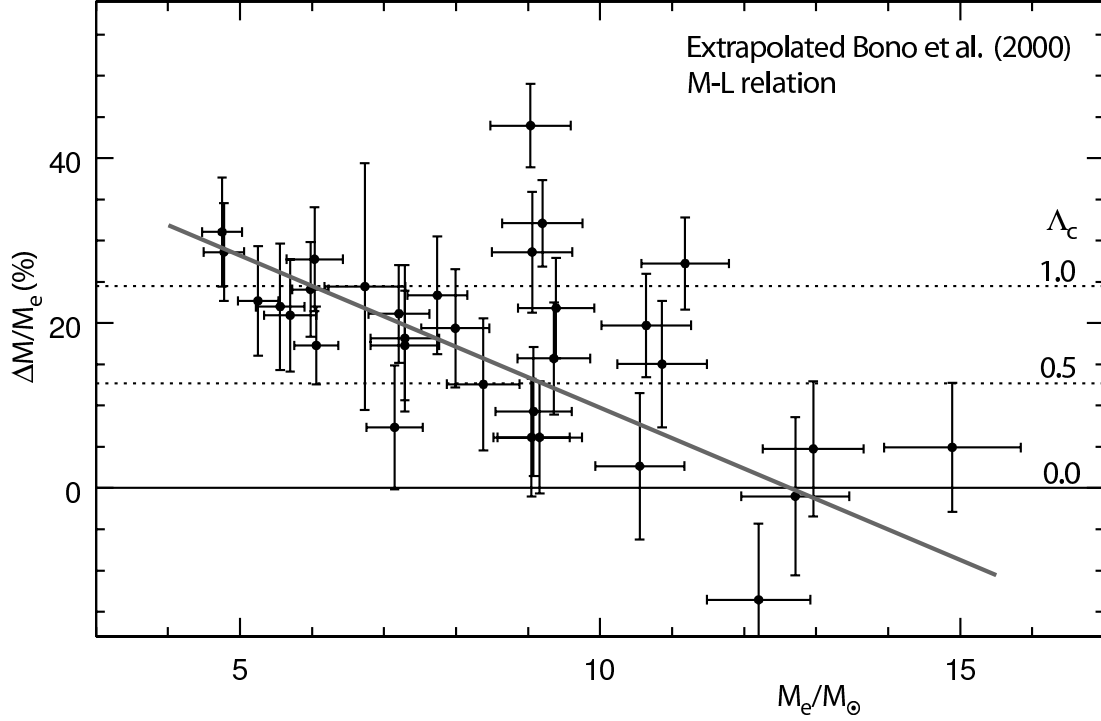
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Fig. 1.— Here we show the mass discrepancy as a function of mass. $\Delta M/M_E$ expresses the mass discrepancy and is equivalent to the difference between the pulsation mass, M_P, and classical evolutionary mass, M_E (i.e. M_E does not incorporate convective core overshoot), normalised by M_E. Here the extrapolated mass-luminosity relation of Bono et al. (2000a) was used (the dashed line in Figure 2, see text for details). Note the mass discrepancy vanishes at higher masses. Overlaid are the locii of models that incorporate mild ($\Lambda = 0.5$; Girardi et al. (2000)) and moderate ($\Lambda = 1.0$; Bressan (2001)) convective core overshoot.
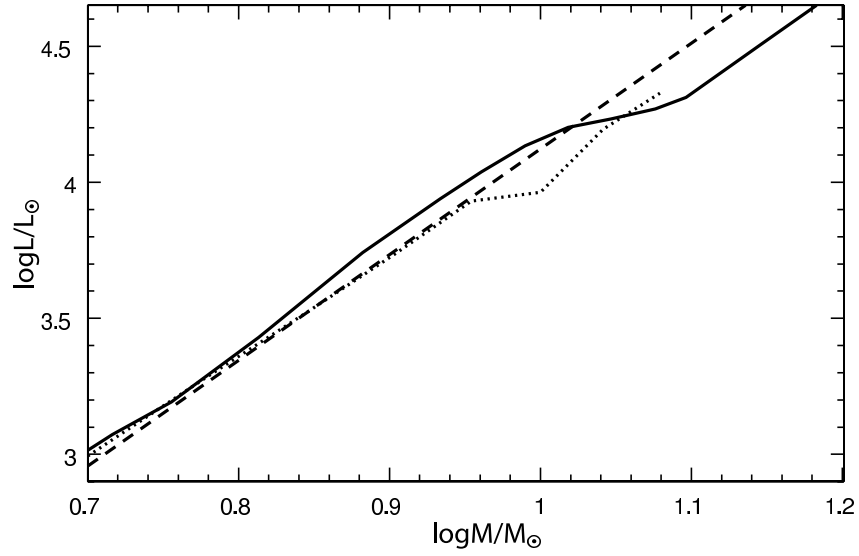
Fig. 2.— The Cepheid mass-luminosity relation used by Caputo et al. (2005) consists of the dashed line (from Caputo et al. Equation 2). The models of Bono et al. (2000a, Z=0.02, Y=0.27 and $\Lambda_c = 0$) without mass-loss are shown by the dotted line. The Bressan et al. (1993) mass-luminosity relationship for Cepheids is shown as the solid line. Note the significant departures from the linear relation of relation of Caputo et al.
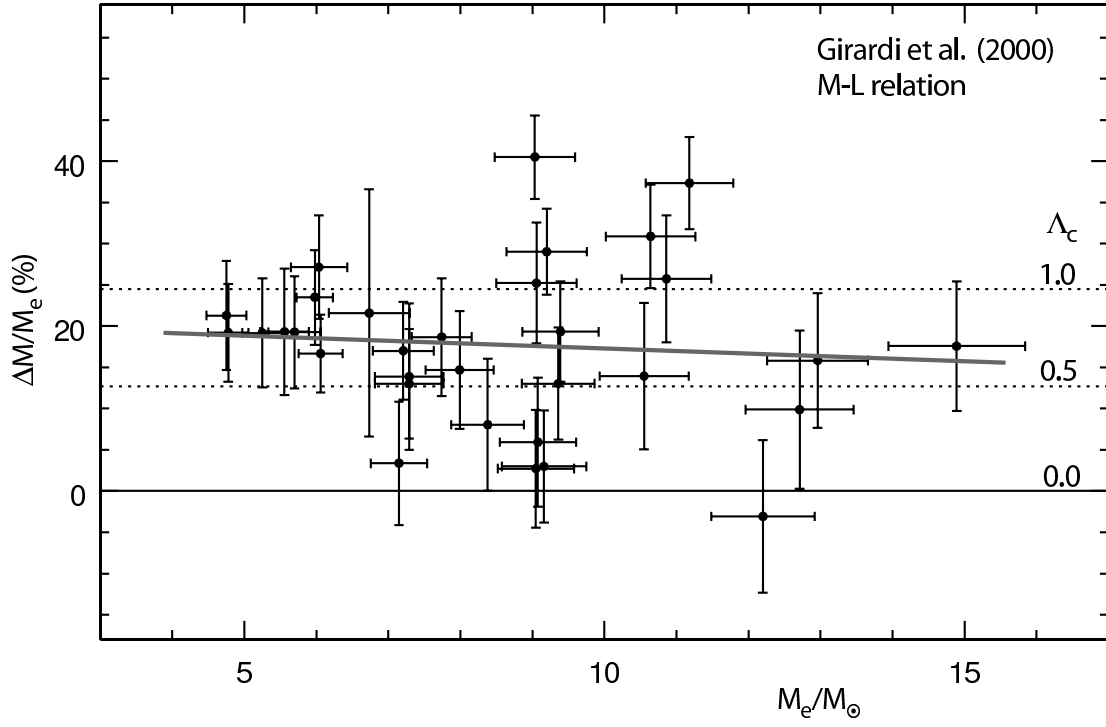
Fig. 3.— As in Figure 1 except here the mass-luminosity relation is due to Bressan et al. (1993). The mass discrepancy shows no significant dependence on mass, but rather a uniform offset of $(17 \pm 5)\%$ (see text for details).